



# Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source

S.S. Das<sup>a,\*</sup>, A. Satapathy<sup>b</sup>, J.K. Das<sup>c</sup>, J.P. Panda<sup>d</sup>

<sup>a</sup> Department of Physics, KBDV College, Nirakarpur, Khurda 752 019, Orissa, India

<sup>b</sup> Department of Physics, ABIT, PMCA, CDA Sector-I, Bidanasi, Cuttack 753 014, Orissa, India

<sup>c</sup> Department of Physics, Stewart Science College, Mission Road, Cuttack 753 001, Orissa, India

<sup>d</sup> Department of Mathematics, Synergy Institute of Engng. & Tech., Dhenkanal 759 001, Orissa, India

## ARTICLE INFO

### Article history:

Received 4 July 2007

Received in revised form 1 February 2009

Accepted 24 April 2009

Available online 16 September 2009

### Keywords:

MHD flow

Mass transfer

Heat transfer

Porosity

Oscillatory suction

Heat source

## ABSTRACT

This paper considers the effect of mass transfer on free convective flow and heat transfer of a viscous incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependant permeability and oscillatory suction in presence of a transverse magnetic field and heat source. The solutions for velocity field, temperature field and concentration distribution are obtained using perturbation technique. The effects of the flow parameters such as magnetic parameter  $M$ , Grashof number for heat and mass transfer  $G_r, G_c$ , porosity parameter  $K_p$ , Prandtl number  $P_r$ , Schmidt number  $S_c$ , frequency parameter  $\omega$  and heat source parameter  $S$  on the velocity, temperature and concentration distribution of the flow field and the skin friction, heat flux and the rate of mass transfer are studied analytically and presented with the aid of figures and tables. It is observed that the magnetic parameter and the Schmidt number retard the velocity of the flow field while the Grashof number for heat and mass transfer, the porosity parameter and the heat source parameter have accelerating effect on the velocity of the flow field at all points. Further, the Prandtl number reduces the temperature and the Schmidt number diminishes the concentration distribution of the flow field at all points. The skin friction coefficients  $\tau_0$  and  $\tau$  increase due to increase in  $G_r, G_c$  and  $K_p$  while decrease due to increase in  $S_c, M, \omega$  and  $P_r$ . Further, the rate of mass transfer  $S_h$  increases due to increase in  $S_c$  while an increase in  $\omega$  results a decrease in  $S_h$ .

© 2009 Elsevier Ltd. All rights reserved.

## 1. Introduction

The phenomenon of MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied applications in science and technology. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo- machinery and in aerospace technology. Such flows arise either due to unsteady motion of boundary or boundary temperature. Besides, unsteadiness may also be due to oscillatory free stream velocity or temperature. In natural processes and industrial applications many transport processes exist where transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is also very common in chemical process industries such as food processing and polymer production.

Several researchers have analyzed the free convection and mass transfer flow of a viscous fluid through porous medium. In their studies, the permeability of the porous medium is assumed to be constant while the porosity of the medium may not necessarily be constant because the porous material containing the fluid is a non-homogeneous medium. Therefore, the permeability of the porous medium may not necessarily be a constant. In light of these facts, Gebhart and Pera [1] showed the nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Gersten and Gross [2] have discussed the flow and heat transfer along a plane wall with periodic suction. Soundalgekar and Gupta [3] investigated the effect of free convection on oscillatory flow past an infinite vertical plate with variable suction and constant heat flux. Georgantopolous et al. [4] have estimated the effect of mass transfer on free convective hydromagnetic oscillatory flow past an infinite vertical porous plate. Hayat et al. [5] have reported the periodic unsteady flows of a non-Newtonian fluid. Kim [6] studied the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction.

\* Corresponding author.

E-mail address: [drssd2@yahoo.com](mailto:drssd2@yahoo.com) (S.S. Das).

**Nomenclature**

$C$	species concentration	$v_0$	constant suction velocity
$C$	non-dimensional species concentration	$y'$	distance along $y$ -axis
$D$	molecular diffusivity	$y$	non-dimensional distance along $y$ -axis
$G_c$	Grashof number for mass transfer	<i>Greek symbols</i>	
$G_r$	Grashof number for heat transfer	$\beta$	volumetric coefficient of expansion for heat transfer
$g$	acceleration due to gravity	$\beta^*$	volumetric coefficient of expansion with species concentration
$K'$	permeability of the medium	$\varepsilon$	a small positive constant ( $\ll 1$ )
$K_p$	permeability/porosity parameter	$\rho$	density of the fluid
$k$	thermal diffusivity	$\nu$	kinematic coefficient of viscosity
$M$	magnetic parameter	$\omega'$	frequency of oscillation
$N_{ii}$	Nusselt number	$\omega$	non-dimensional frequency of oscillation
$P_r$	Prandtl number	$\tau$	skin friction
$S$	heat source parameter	$\sigma$	electrical conductivity
$S_c$	Schmidt number	<i>Subscripts</i>	
$S_h$	Sherwood number	w	condition on the porous plate
$T'$	temperature of the fluid	$\infty$	condition far away from the plate
$T$	non-dimensional temperature	<i>Superscripts</i>	
$t'$	time	'	differentiation with respect to $y$ (used in Eqs. (11)–(16))
$t$	non-dimensional time		
$u'$	velocity component along $x$ -axis		
$u$	non-dimensional velocity component along $x$ -axis		
$v(t')$	suction velocity		

The problem of three dimensional free convective flow and heat transfer through a porous medium with periodic permeability has been discussed by Singh and Sharma [7]. Singh and his co-workers [8] have analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Asghar et al. [9] have reported the flow of a non-Newtonian fluid induced due to the oscillations of a porous plate. Bathul [10] discussed the heat transfer in a three dimensional viscous flow over a porous plate moving with a harmonic disturbance. Singh and Gupta [11] have investigated the MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillatory porous plate in the slip-flow regime with mass transfer. Das and his co-workers [12] analyzed the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction employing numerical methods. Ogulu and Prakash [13] considered heat transfer to unsteady magneto hydrodynamic flow past an infinite vertical moving plate with variable suction. Das et al. [14] discussed the free convective and mass transfer flow of a viscous fluid past an infinite vertical porous plate through a porous medium in presence of source/sink with constant suction and heat flux.

In the studies mentioned above, the oscillatory suction velocity in presence of time dependent viscosity along with the influence of uniform magnetic field with heat source have not been discussed while such flows are very common in geophysical and astrophysical problems and also in soil sciences. Therefore, the objective of the present study is to analyze the effects of permeability variation and oscillatory suction velocity in free convective and mass transfer flow of a viscous incompressible fluid past an infinite vertical porous plate through a porous medium when the plate is subjected to a time dependent suction velocity normal to the plate in the presence of a uniform transverse magnetic field with heat source. The solutions for velocity field, temperature field and concentration distribution are obtained using perturbation technique. The results obtained are discussed for Grashof number,  $G_r > 0$  corresponding to cooling of the plate. The effects of the flow parameters such as magnetic parameter  $M$ , Grashof number for heat and mass transfer  $G_r, G_c$ , porosity parameter  $K_p$ , Prandtl number  $P_r$ , Schmidt number  $S_c$ ,

heat source parameter  $S$  and frequency parameter  $\omega$  on the velocity, temperature and concentration distribution of the flow field have been studied analytically and presented graphically. Further, the effects of the flow parameters on skin friction, heat flux and the rate of mass transfer have been discussed with the help of tables.

**2. Formulation of the problem**

Consider the unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in a porous medium of time dependent permeability and oscillatory suction in presence of a transverse magnetic field. Let  $x'$ -axis be along the plate in the direction of flow and  $y'$ -axis normal to it. Let us assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. The basic flow in the medium is therefore entirely due to the buoyancy force caused by temperature difference between the wall and the medium. Initially, at  $t' \leq 0$ , the plate as well as the fluid are assumed to be at the same temperature and the concentration of the species is very low so that the Soret and Dofour effects are neglected (Gebhart and Pera [1]). When  $t' > 0$ , the temperature of the plate is instantaneously raised (or lowered) to  $T'_w$  and the concentration of the species is raised (or lowered) to  $C'_w$ .

The permeability of the porous medium is assumed to be of the form

$$K'(t') = K_p(1 + \varepsilon e^{i\omega' t'}) \quad (1)$$

and the suction velocity is assumed to be

$$v(t') = -v_0(1 + \varepsilon e^{i\omega' t'}), \quad (2)$$

where  $v_0 > 0$  and  $\varepsilon \ll 1$  is a positive constant. Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations for momentum, energy and concentration in non-dimensional form are:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial u}{\partial y} = G_r T + G_c C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_p(1 + \varepsilon e^{i\omega t})} - M^2 u, \quad (3)$$

$$\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} + ST, \quad (4)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2}. \quad (5)$$

The necessary boundary conditions for this flow are:

$$\begin{aligned} u = 0, \quad T = 1 + \varepsilon e^{i\omega t}, \quad C = 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0, \\ u \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (6)$$

We introduced the following non-dimensional quantities in the above equations:

$$\begin{aligned} y = \frac{v_0 y'}{v}, \quad t = \frac{v_0^2 t'}{4\nu}, \quad \omega = \frac{4\nu\omega'}{v_0^2}, \quad u = \frac{u'}{v_0}, \quad T = \frac{T' - T_\infty}{T_w - T_\infty}, \\ C = \frac{C' - C_\infty}{C_w - C_\infty}, \quad S = \frac{\nu S'}{v_0^2}, \quad K_p = \frac{\nu^2 K'}{v_0^2}, \quad G_r = \frac{\nu g \beta (T_w - T_\infty)}{v_0^3}, \\ G_c = \frac{\nu g \beta^* (C_w - C_\infty)}{v_0^3}, \quad S_c = \frac{\nu}{D}, \quad P_r = \frac{\nu}{k}, \quad M^2 = \frac{\sigma B_0^2 \nu}{\rho v_0^2}. \end{aligned} \quad (7)$$

### 3. Method of solution

In order to reduce the system of partial differential Eqs. (3)–(5) to a system of ordinary differential equations in the non-dimensional form, we assume the following for velocity, temperature and concentration distribution of the flow field as the amplitude  $\varepsilon (\ll 1)$  of the permeability variation is very small.

$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t}, \quad (8)$$

$$T(y, t) = T_0(y) + \varepsilon T_1(y) e^{i\omega t}, \quad (9)$$

$$C(y, t) = C_0(y) + \varepsilon C_1(y) e^{i\omega t}. \quad (10)$$

Substituting Eqs. (8)–(10) into Eqs. (3)–(5) and equating the harmonic and non-harmonic terms, we obtain

$$u_0'' + u_0' - a_1 u_0 = -G_r T_0 - G_c C_0, \quad (11)$$

$$u_1'' + u_1' - a_2 u_1 = -G_r T_1 - G_c C_1 - u_0' - \frac{1}{K_p} u_0, \quad (12)$$

$$T_0'' + P_r T_0' - S T_0 = 0, \quad (13)$$

$$T_1'' + P_r T_1' - \frac{i\omega}{4} P_r T_1 - S T_1 = -P_r T_0', \quad (14)$$

$$C_0'' + S_c C_0' = 0, \quad (15)$$

$$C_1'' + S_c C_1' - \frac{i\omega}{4} S_c C_1 = -S_c C_0', \quad (16)$$

where  $a_1 = M^2 + \frac{1}{K_p}$ ,  $a_2 = a_1 + \frac{\omega}{4}$ .

The boundary conditions now reduce to

$$\begin{aligned} u_0 = u_1 = 0, \quad T_0 = T_1 = 1, \quad C_0 = C_1 = 1 \quad \text{at } y = 0 \\ u_0 = u_1 \rightarrow 0, \quad T_0 = T_1 \rightarrow 0, \quad C_0 = C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (17)$$

Solving Eqs. (11)–(16) under boundary conditions (17), we get

$$\begin{aligned} u(y, t) = & \frac{G_r(e^{-m_1 y} - e^{-m_5 y})}{(m_5 - m_1)(m_1 + m_6)} + \frac{G_c(e^{-S_c y} - e^{-m_5 y})}{(m_5 - S_c)(m_6 + S_c)} e^{-S_c y} \\ & + \varepsilon \left[ \frac{G_r(e^{-m_3 y} - e^{-m_7 y})}{(m_7 - m_3)(m_3 + m_8)} \left( 1 + \frac{P_r m_1}{(m_3 - m_1)(m_1 + m_4)} \right) \right. \\ & - \frac{G_r P_r m_1 (e^{-m_1 y} - e^{-m_7 y})}{(m_7 - m_1)(m_1 + m_8)(m_3 - m_1)(m_1 + m_4)} \\ & - \frac{G_r \left( m_1 - \frac{1}{K_p} \right) (e^{-m_1 y} - e^{-m_7 y})}{(m_5 - m_1)(m_1 + m_6)(m_7 - m_1)(m_1 + m_8)} \\ & + \frac{G_r \left( m_5 - \frac{1}{K_p} \right) (e^{-m_5 y} - e^{-m_7 y})}{(m_5 - m_1)(m_1 + m_6)(m_7 - m_5)(m_5 + m_8)} \\ & + \frac{G_c \left( 1 - i \frac{4S_c}{\omega} \right) (e^{-m_2 y} - e^{-m_7 y})}{(m_7 - m_2)(m_2 + m_8)} \\ & \left. - \frac{G_c \left( -S_c + \frac{1}{K_p} \right) (e^{-S_c y} - e^{-m_7 y})}{(m_5 - S_c)(m_6 + S_c)(m_7 - S_c)(m_8 + S_c)} \right. \\ & \left. - i \frac{4S_c G_c (e^{-S_c y} - e^{-m_7 y})}{\omega (m_7 - S_c)(m_8 + S_c)} + \frac{G_c \left( -m_5 + \frac{1}{K_p} \right)}{(m_5 - S_c)(m_6 + S_c)} \right. \\ & \left. \times \frac{(e^{-m_5 y} - e^{-m_7 y})}{(m_7 - m_5)(m_5 + m_8)} \right] e^{i\omega t}, \end{aligned} \quad (18)$$

$$T(y, t) = e^{-m_1 y} + \varepsilon \left[ e^{-m_3 y} + \frac{P_r m_1 (e^{-m_3 y} - e^{-m_1 y})}{(m_3 - m_1)(m_1 + m_4)} \right] e^{i\omega t}, \quad (19)$$

$$C(y, t) = e^{-S_c y} + \varepsilon \left[ \left( 1 - i \frac{4S_c}{\omega} \right) e^{-m_2 y} + i \frac{4S_c}{\omega} e^{-S_c y} \right] e^{i\omega t}. \quad (20)$$

Separating the real and imaginary parts from Eqs. (18)–(20) and taking only the real parts as they have physical significance, the velocity, temperature and concentration distribution of the flow field can be expressed in fluctuating parts as given below.

$$u(y, t) = u_0(y) + \varepsilon (M_r \cos \omega t - M_i \sin \omega t), \quad (21)$$

$$T(y, t) = T_0(y) + \varepsilon (K_r \cos \omega t - K_i \sin \omega t), \quad (22)$$

$$C(y, t) = C_0(y) + \varepsilon (L_r \cos \omega t - L_i \sin \omega t). \quad (23)$$

Now the expressions for transient velocity, temperature and concentration distribution of the flow field for  $\omega t = \frac{\pi}{2}$  are,

$$u\left(y, \frac{\pi}{2\omega}\right) = u_0(y) - \varepsilon M_i, \quad (24)$$

$$T\left(y, \frac{\pi}{2\omega}\right) = T_0(y) - \varepsilon K_i, \quad (25)$$

$$C\left(y, \frac{\pi}{2\omega}\right) = C_0(y) - \varepsilon L_i. \quad (26)$$

#### 3.1. Skin friction

The skin friction at the plate in terms of amplitude and phase is given by,

$$\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \tau_0 + \varepsilon |N| \cos(\omega t + \alpha), \quad (27)$$

where  $\tau_0 = \frac{G_r}{(m_1 + m_6)} + \frac{G_c}{(m_6 + S_c)}$ ,

$$N = N_r + iN_i = \left( \frac{du_1}{dy} \right)_{y=0}, \quad \tan \alpha = \frac{N_i}{N_r}. \quad (28)$$

#### 3.2. Rate of heat transfer

The rate of heat transfer, i.e., heat flux at the plate ( $N_u$ ) in terms of amplitude and phase is given by,

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = m_1 + \varepsilon|R|\text{Cos}(\omega t + \delta), \tag{29}$$

$$\text{where } R = R_r + iR_i = -\left(\frac{dT_1}{dy}\right)_{y=0}, \quad \tan \delta = \frac{R_i}{R_r}. \tag{30}$$

3.3. Rate of mass transfer

The mass transfer coefficient, i.e., the Sherwood number ( $S_h$ ) at the plate in terms of amplitude and phase is given by,

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = S_c + \varepsilon|Q|\text{Cos}(\omega t + \gamma), \tag{31}$$

$$\text{where } Q = Q_r + iQ_i = -\left(\frac{dC_1}{dy}\right)_{y=0}, \quad \tan \gamma = \frac{Q_i}{Q_r}. \tag{32}$$

The constants  $a_3, a_4, b_1-b_{10}, A_1-A_{12}, B_1-B_{12}, N_1-N_{18}, m_1-m_8, M_r, M_i, L_r, L_i, N_r, N_i, R_r, R_i, Q_r$  and  $Q_i$  appearing in the text are appended before the reference section of the text.

4. Results and discussion

The problem of free convective flow with heat and mass transfer of a viscous incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependant permeability and oscillatory suction in presence of a transverse magnetic field and heat source has been considered. The solutions for velocity field and temperature field are obtained using perturbation technique. The effects of the flow parameters such as magnetic parameter  $M$ , Grashof number for heat and mass transfer  $G_r, G_c$ , porosity parameter  $K_p$ , heat source parameter  $S$ , Prandtl number  $P_r$  and frequency parameter  $\omega$  on the velocity field have been studied analytically and presented with the help of Figs. 1–3. The effects of the flow parameters on the temperature field and concentration distribution have been presented in Figs. 4 and 5, respectively. Further, the effects of the flow parameters on skin friction, heat flux and the rate of mass transfer have been discussed with the help of Tables 1–4. The computational work of the problem is carried on using FORTRAN programme.

4.1. Velocity field ( $u$ )

The velocity of the flow field varies vastly with the variation of the flow parameters such as magnetic parameter  $M$ , Grashof number for heat and mass transfer  $G_r, G_c$ , porosity parameter  $K_p$ , heat

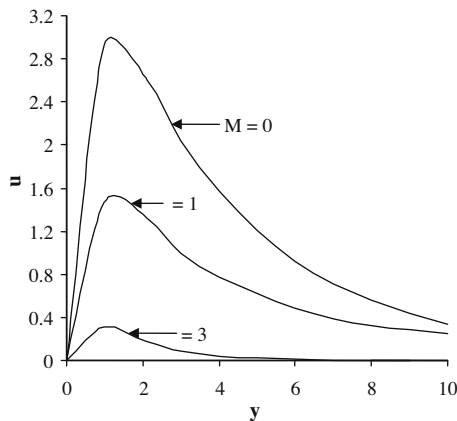


Fig. 1. Effect of  $M$  on velocity profiles with  $G_r = 10, G_c = 10, S_c = 0.22, K_p = 10, S = 0, \varepsilon = 0.002, P_r = 0.71, \omega = 5$  and  $\omega t = \pi/2$ .

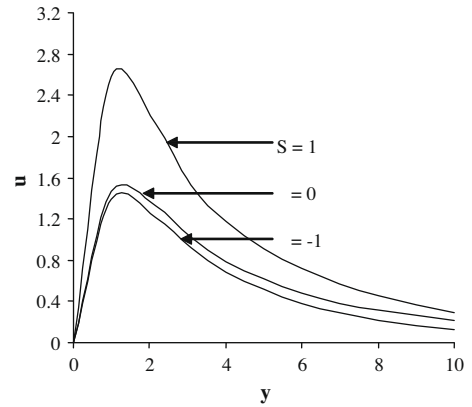


Fig. 2. Effect of  $S$  on velocity profiles with  $G_r = 10, G_c = 10, S_c = 0.22, M = 1, K_p = 10, \varepsilon = 0.002, P_r = 0.71, \omega = 5$  and  $\omega t = \pi/2$ .

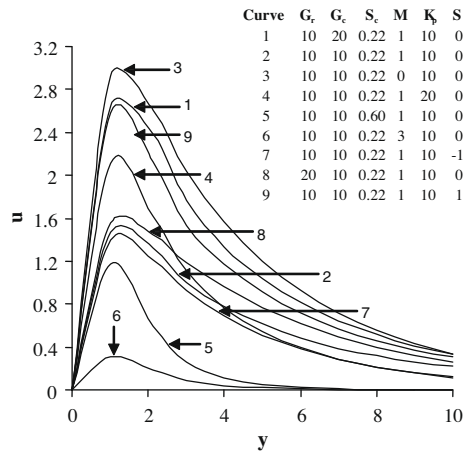


Fig. 3. Effect of different parameters on velocity profiles with  $\varepsilon = 0.002, P_r = 0.71, \omega = 5$  and  $\omega t = \pi/2$ .

source parameter  $S$  and Prandtl number  $P_r$ . The effects of these parameters on the flow field have been presented in Figs. 1–3.

4.1.1. Effect of magnetic parameter ( $M$ )

Fig. 1 depicts the effect of magnetic parameter  $M$  on the velocity profiles of the flow field keeping other parameters of the flow field constant. The curve with magnetic parameter,  $M = 0$  corresponds to non-MHD flow and in other two curves the magnetic parameter is taken in increasing order. The magnetic parameter is found to decelerate the velocity of the flow field to an appreciable amount due to the magnetic pull of the Lorentz force acting on the flow field. In case of Singh et al. [8], the above parameter shows reverse effect.

4.1.2. Effect of heat source parameter ( $S$ )

The effect of heat source parameter  $S$  on the velocity profiles of the flow field is shown in Fig. 2 for  $S = 0, 1$  and  $-1$  keeping other parameters of the flow field constant. The curve with  $S = 0$  corresponds to the no heat source case. This curve is in between curves with  $S = -1$  and  $S = 1$ . Comparing the three curves of the figure, it is observed that the heat source parameter enhances the velocity at all points of the flow field. The effect of heat source parameter ( $S > 0$ ) on velocity profiles is very much significant compared to the heat sink parameter ( $S < 0$ ).

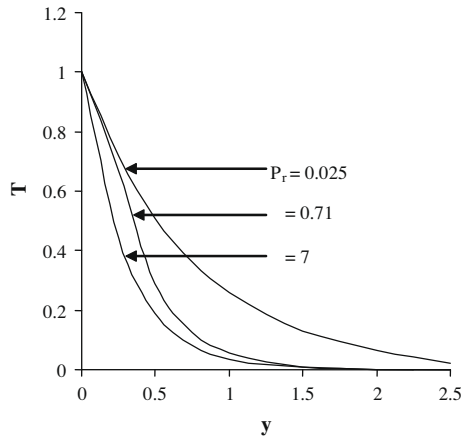


Fig. 4. Effects  $P_r$  on temperature profile with  $\varepsilon = 0.002, \omega = 5$  and  $\omega t = \pi/2$ .

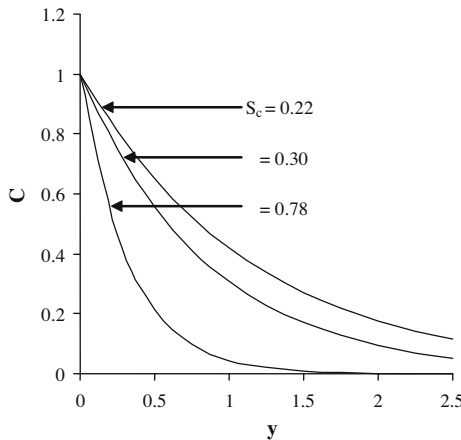


Fig. 5. Effects  $S_c$  on concentration distribution with  $\varepsilon = 0.002, \omega = 5$  and  $\omega t = \pi/2$ .

Table 1  
Values of amplitude  $|N|$  and phase  $\tan \alpha$  for skin friction coefficient due to the variation of  $G_r, G_c, S_c, M, K_p, \omega$  and  $P_r$  with  $S = 1$ .

$G_r$	$G_c$	$S_c$	$M$	$K_p$	$\omega$	$P_r$	$ N $	$\tan \alpha$
5.0	2.0	0.22	0.5	10.0	5.0	0.71	26.7120	-0.2179
10.0	2.0	0.22	0.5	10.0	5.0	0.71	59.5174	-0.1883
5.0	4.0	0.22	0.5	10.0	5.0	0.71	25.6200	-0.2832
5.0	2.0	0.66	0.5	10.0	5.0	0.71	27.1928	-0.3172
5.0	2.0	0.22	1.0	10.0	5.0	0.71	15.3142	0.3866
5.0	2.0	0.22	0.5	20.0	5.0	0.71	24.9037	-0.3683
5.0	2.0	0.22	0.5	10.0	10.0	0.71	10.1276	-0.3819
5.0	2.0	0.22	0.5	10.0	5.0	7.0	1.8271	-1.6693

Table 2  
Values of skin friction coefficients  $\tau_0$  and  $\tau$  due to the variation of  $G_r, G_c, S_c, M, K_p, \omega$  and  $P_r$  with  $S = 1, \varepsilon = 0.002$  and  $\omega t = \pi/2$ .

$G_r$	$G_c$	$S_c$	$M$	$K_p$	$\omega$	$P_r$	$\tau_0$	$\tau$
5.0	2.0	0.22	0.5	10.0	5.0	0.71	7.0010	7.0123
10.0	2.0	0.22	0.5	10.0	5.0	0.71	9.9583	9.9782
5.0	4.0	0.22	0.5	10.0	5.0	0.71	11.0447	11.059
5.0	2.0	0.66	0.5	10.0	5.0	0.71	5.0972	5.1137
5.0	2.0	0.22	1.0	10.0	5.0	0.71	4.6740	4.6639
5.0	2.0	0.22	0.5	20.0	5.0	0.71	7.3487	7.3613
5.0	2.0	0.22	0.5	10.0	10.0	0.71	7.0008	7.0005
5.0	2.0	0.22	0.5	10.0	5.0	7.0	4.7180	4.7149

Table 3  
Values of amplitude  $|R|$ , phase  $\tan \delta$  and rate of heat transfer  $N_u$  for different values of  $P_r$  and  $\omega$  with  $S = 1, \varepsilon = 0.002$  and  $\omega t = \pi/2$ .

$P_r$	$\omega$	$ R $	$\tan \delta$	$N_u$
0.71	10.0	1.7605	-0.9551	1.4186
0.025	10.0	1.0089	-0.0253	1.0126
7.0	10.0	19.2757	-7.8378	7.1783
9.0	10.0	25.4758	-9.9388	9.1605
11.4	10.0	32.8685	-12.4716	11.553
0.71	15.0	2.0308	0.0171	1.4161

Table 4  
Values of amplitude  $|Q|$ , phase  $\tan \gamma$  and rate of mass transfer  $S_h$  for different values of  $S_c$  and  $\omega$  with  $\varepsilon = 0.002$  and  $\omega t = \pi/2$ .

$S_c$	$\omega$	$ Q $	$\tan \gamma$	$S_h$
0.22	10.0	0.6594	0.3117	0.2196
0.30	10.0	0.8086	0.3374	0.2995
0.60	10.0	1.3444	0.3772	0.5990
0.66	10.0	1.4526	0.3796	0.6590
0.78	10.0	1.6731	0.3813	0.7788
0.22	15.0	1.7718	0.3453	0.2195

4.1.3. Effect of different parameters ( $G_r, G_c, S_c, K_p$ )

Fig. 3 presents the effect of Grashof number for heat and mass transfer  $G_r, G_c$ , Schmidt number  $S_c$ , magnetic parameter  $M$ , porosity parameter  $K_p$  and heat source parameter  $S$  on the velocity profiles of the flow field. As the effects of magnetic parameter  $M$  and the heat source parameter  $S$  are already discussed in Figs. 1 and 2, it is needless to mention their effects again. We now wish to discuss the effects of  $G_r, G_c, S_c$  and  $K_p$  on the velocity profiles of the flow field. Curves (2) and (8) of the figure depict the effect of  $G_r$  on the velocity of the flow field. The effect of  $G_r$  is to enhance the velocity of the flow field at all points. Comparing the curves (1) and (2) of the figure, it is observed that the Grashof number for mass transfer  $G_c$  accelerates the velocity of the flow field at all points. Curves (2) and (5) describe the effect of Schmidt number  $S_c$  on the velocity profiles of the flow field which reveal that the presence of heavier diffusing species has a retarding effect on the velocity of the flow field. The effect of porosity parameter  $K_p$  on the velocity field is shown by the curves (2) and (4). The porosity parameter is found to enhance the velocity at all points of the flow field. The effect of the above parameters reverses in case of Singh et al. [8]. This remarkable feature of the velocity profiles in our investigation is due to the presence of heat source in the flow field.

4.2. Temperature field (T)

The temperature field is found to change more or less with the variation of the Prandtl number  $P_r$ . Fig. 4 is a plot of non-dimensional temperature and distance for three different values of the Prandtl number. Curve with  $P_r = 0.025$  corresponds to mercury,  $P_r = 0.71$  for air and  $P_r = 7$  for water at 20 °C. The temperature of the flow field diminishes as the Prandtl number increases. Higher the Prandtl number, the sharper is the reduction in the temperature of the flow field. The temperature profiles are in good agreement with the results obtained in case of Singh et al. [8].

4.3. Concentration distribution (C)

The variation of the concentration distribution of the flow field with the diffusion of the foreign mass such as hydrogen ( $S_c = 0.22$ ), helium ( $S_c = 0.30$ ) and ammonia ( $S_c = 0.78$ ) is shown in Fig. 5. The concentration distribution decreases at all points of the flow field with the increase of the Schmidt number. This shows that the hea-



vier the diffusing species have a greater retarding effect on the concentration distribution of the flow field. The concentration profiles closely agree with those of Singh et al. [8].

#### 4.4. Skin friction

The numerical values of skin friction coefficient  $\tau$  in terms of amplitude  $|N|$  and phase  $\tan\alpha$  for different values of  $G_r, G_c, S_c, M, K_p, \omega$  and  $P_r$  with  $S=1$  corresponding to cooling of the plate ( $G_r > 0$ ) are entered in Table 1. It is observed that an increase in  $G_r$  and  $S_c$  leads to an increase in the value of amplitude  $|N|$  while an increase in  $G_c, M, K_p, \omega$  and  $P_r$  leads to a decrease in the value of  $|N|$ . The value of phase  $\tan\alpha$  increases due to increase in  $G_r, M$  while decreases due to increase in  $G_c, S_c, K_p, \omega$  and  $P_r$ . One interesting observation of this table is that there is a phase lag in skin friction due to increase in  $G_c, S_c, K_p, \omega$  and  $P_r$  while there is a phase lead for magnetic parameter  $M$  and Grashof number for heat transfer  $G_r$ . The effects of all the parameters except  $G_r$  closely agree with the results of Singh et al. [8], while  $G_r$  shows the reverse effect. This remarkable feature of  $G_r$  is due to the presence of heat source in the flow field.

In Table 2, we present the values of skin friction coefficient  $\tau_0$  due to steady part of the velocity and skin friction coefficient  $\tau$  due to cooling of the plate ( $G_r > 0$ ) with  $S=1$ ,  $\varepsilon=0.002$  and  $\omega t = \pi/2$  for different values of  $G_r, G_c, S_c, M, K_p, \omega$  and  $P_r$ , respectively. It is observed that the skin friction coefficients  $\tau_0$  and  $\tau$  increase due to increase in  $G_r, G_c$  and  $K_p$  while decrease due to increase in  $S_c, M, \omega$  and  $P_r$  which is just the reverse of the results obtained in case of Singh et al. [8]. This is due to the dominant role of the heat source parameter in the flow field over the cooling of the plate.

#### 4.5. Rate of heat transfer

Table 3 presents the numerical values of amplitude  $|R|$ , phase  $\tan\delta$  and rate of heat transfer in terms of Nusselt number  $N_u$  due to variations in the values of Prandtl number  $P_r$  and frequency  $\omega$  with  $S=1$ ,  $\varepsilon=0.002$  and  $\omega t = \pi/2$ . It is observed that amplitude  $|R|$  and rate of heat transfer  $N_u$  are least for mercury and highest for water at 4 °C while reverse effect is observed for phase  $\tan\delta$ . The values of the amplitude and the rate of heat transfer for air and water lies between the said two values while the value of  $\tan\delta$  is more for mercury in comparison to air and water. An increase in  $\omega$  results in an increase in amplitude  $|R|$  and the phase  $\tan\delta$  and a decrease in the rate of heat transfer  $N_u$ . Our results are in good agreement with the results of Singh et al. [8].

#### 4.6. Rate of mass transfer

Table 4 presents the numerical values of amplitude  $|Q|$ , phase  $\tan\gamma$  and rate of mass transfer in terms of Sherwood number  $S_h$  with  $\varepsilon=0.002$  and  $\omega t = \pi/2$  due to variations in the values of  $S_c$  for hydrogen, helium, water vapour, oxygen and ammonia. It is observed that the values of amplitude  $|Q|$ , phase  $\tan\gamma$  and the rate of mass transfer  $S_h$  increase due to increase in  $S_c$  while an increase in  $\omega$  results in an increase in  $|Q|$  and  $\tan\gamma$  and a decrease in  $S_h$ . The effect of  $S_c$  on phase  $\tan\gamma$  reverses in case of Singh et al. [8].

### 5. Conclusions

The above analysis brings out the following results of physical interest on the velocity, temperature and concentration distribution of the flow field.

1. The magnetic parameter  $M$  retards the velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field.
2. The effect of heat source parameter  $S$  is to enhance the velocity of the flow field at all points.
3. The Grashof numbers for heat transfer  $G_r$  and mass transfer  $G_c$  accelerate the velocity of the flow field.
4. The Schmidt number  $S_c$  has a retarding effect on the velocity of the flow field. Heavier the diffusing species, the more is the retarding effect on the fluid velocity.
5. The effect of porosity parameter  $K_p$  is to enhance the velocity of the flow field at all points.
6. The Prandtl number  $P_r$  reduces the temperature of the flow field at all points. Higher the Prandtl number, the sharper is the reduction in temperature of the flow field.
7. The concentration distribution of the flow field decreases at all points as the Schmidt number  $S_c$  increases. This means the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field.
8. A growing  $G_r$  or  $S_c$  leads to an increase in the value of amplitude of skin friction  $|N|$  while a growing  $G_c$  or  $M$  or  $K_p$  or  $\omega$  or  $P_r$  leads to a decrease in the value of  $|N|$ . The phase  $\tan\alpha$  increases due to increase in  $G_r$  or  $M$  while decreases due to increase in  $G_c$  or  $S_c$  or  $K_p$  or  $\omega$  or  $P_r$ . Further, there is a phase lag in skin friction due to increase in  $G_c, S_c, K_p, \omega$  and  $P_r$  and a phase lead for  $M$  and  $G_r$ .
9. The skin friction coefficients  $\tau_0$  and  $\tau$  increase due to increase in  $G_r, G_c$  and  $K_p$  while decrease due to increase in  $S_c, M, \omega$  and  $P_r$ .
10. The amplitude of heat transfer  $|R|$  and rate of heat transfer  $N_u$  are least for mercury and highest for water at 4 °C while the effect reverses for phase  $\tan\delta$ . The values of the amplitude and the rate of heat transfer for air and water lie between the said two values while the value of  $\tan\delta$  is more for mercury in comparison to air and water. An increase in  $\omega$  results in an increase in amplitude  $|R|$  and the phase  $\tan\delta$  and a decrease in the rate of heat transfer  $N_u$ .
11. The amplitude of mass transfer  $|Q|$ , phase  $\tan\gamma$  and the rate of mass transfer  $S_h$  increase due to increase in  $S_c$  while an increase in  $\omega$  results in an increase in  $|Q|$  and  $\tan\gamma$  and a decrease in  $S_h$ .

### Appendix A

We list below the constants appearing in the text. They are arranged in accordance with their appearance in the text.

$$m_1 = \frac{1}{2} \left[ P_r + \sqrt{P_r^2 + 4S} \right], \quad m_2 = \frac{1}{2} \left[ S_c + \sqrt{S_c^2 + i\omega S_c} \right] = A_1 + iB_1,$$

$$m_3 = \frac{1}{2} \left[ P_r + \sqrt{P_r^2 + 4S + i\omega P_r} \right] = A_2 + iB_2,$$

$$m_4 = \frac{1}{2} \left[ -P_r + \sqrt{P_r^2 + 4S + i\omega P_r} \right] = A_3 + iB_3,$$

$$m_5 = \frac{1}{2} \left[ 1 + \sqrt{1 + 4 \left( M^2 + \frac{1}{K_p} \right)} \right] = \frac{1}{2} \left[ 1 + \sqrt{1 + 4a_1} \right],$$

$$m_6 = \frac{1}{2} \left[ -1 + \sqrt{1 + 4 \left( M^2 + \frac{1}{K_p} \right)} \right] = \frac{1}{2} \left[ -1 + \sqrt{1 + 4a_1} \right],$$

$$m_7 = \frac{1}{2} \left[ 1 + \sqrt{1 + 4 \left( M^2 + \frac{1}{K_p} + \frac{\omega}{4} \right)} \right] = \frac{1}{2} \left[ 1 + \sqrt{1 + 4a_2} \right] \\ = A_4 + iB_4,$$

$$m_8 = \frac{1}{2} \left[ -1 + \sqrt{1 + 4 \left( M^2 + \frac{1}{K_p} + \frac{\omega}{4} \right)} \right]$$

$$= \frac{1}{2} \left[ -1 + \sqrt{1 + 4a_2} \right] = A_5 + iB_5,$$

$$(m_3 - m_1)(m_1 + m_4) = A_6 + iB_6, \quad (m_7 - m_3)(m_3 + m_8) = A_7 + iB_7,$$

$$(m_7 - m_1)(m_1 + m_8) = A_8 + iB_8, \quad (m_5 - m_1)(m_1 + m_8) = A_9 + iB_9,$$

$$(m_7 - m_5)(m_5 + m_8) = A_{10} + iB_{10}, \quad (m_7 - m_2)(m_2 + m_8) = A_{11} + iB_{11},$$

$$(m_7 - S_c)(m_8 + S_c) = A_{12} + iB_{12}, \quad (m_5 - S_c)(m_6 + S_c) = a_3,$$

$$(m_5 - m_1)(m_1 + m_6) = a_4,$$

$$A_1 = \frac{S_c}{2} + \frac{1}{2} \sqrt{\frac{S_c}{2} \left[ (S_c^2 + \omega^2)^{\frac{1}{2}} + S_c \right]},$$

$$A_2 = \frac{P_r}{2} + \frac{1}{2\sqrt{2}} \left[ \left\{ (P_r^2 + 4S)^2 + \omega^2 P_r^2 \right\}^{\frac{1}{2}} + (P_r^2 + 4S) \right]^{\frac{1}{2}},$$

$$A_3 = -\frac{P_r}{2} + \frac{1}{2\sqrt{2}} \left[ \left\{ (P_r^2 + 4S)^2 + \omega^2 P_r^2 \right\}^{\frac{1}{2}} + (P_r^2 + 4S) \right]^{\frac{1}{2}},$$

$$A_4 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \left\{ (1 + 4a_1)^2 + \omega^2 \right\}^{\frac{1}{2}} + (1 + 4a_1) \right]^{\frac{1}{2}},$$

$$A_5 = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \left\{ (1 + 4a_1)^2 + \omega^2 \right\}^{\frac{1}{2}} + (1 + 4a_1) \right]^{\frac{1}{2}},$$

$$A_6 = (A_2 - m_1)(m_1 + A_3) - B_2^2,$$

$$A_7 = (A_2 + A_5)(A_4 - A_2) + (B_2 + B_5)(B_2 - B_4),$$

$$A_8 = (A_4 - m_1)(m_1 + A_5) - B_4^2,$$

$$A_9 = (m_5 - m_1)(m_1 + A_5), \quad A_{10} = (A_4 - m_5)(m_5 + A_5) - B_4^2,$$

$$A_{11} = (A_4 - A_1)(A_1 + A_5) + (B_1^2 - B_4^2),$$

$$A_{12} = (A_4 - S_c)(S_c + A_5) - B_2^2,$$

$$B_1 = \frac{1}{2} \frac{S_c}{\sqrt{2}} \left[ (S_c^2 + \omega^2)^{\frac{1}{2}} - S_c \right]^{\frac{1}{2}},$$

$$B_2 = B_3 = \frac{1}{2\sqrt{2}} \left[ \left\{ (P_r^2 + 4S)^2 + \omega^2 P_r^2 \right\}^{\frac{1}{2}} - (P_r^2 + 4S) \right]^{\frac{1}{2}},$$

$$B_4 = B_5 = \frac{1}{2\sqrt{2}} \left[ \left\{ (1 + 4a_1)^2 + \omega^2 \right\}^{\frac{1}{2}} - (1 + 4a_1) \right]^{\frac{1}{2}}, \quad B_6 = (A_2 + A_3)B_2,$$

$$B_7 = (B_2 + B_5)(A_4 - A_2) + (A_2 + A_5)(B_4 - B_2), \quad B_8 = (A_4 + A_5)B_4,$$

$$B_9 = B_5(m_5 - m_1), \quad B_{10} = (A_4 + A_5)B_4,$$

$$B_{11} = B_1(A_4 - A_5) + B_4(A_4 + A_5) - 2A_1B_1, \quad B_{12} = B_{10},$$

$$M_r = (N_1 + N_3 + N_5 + N_7 + N_9 + N_{11} + N_{13} + N_{15} + N_{17}),$$

$$M_i = -(N_2 + N_4 + N_6 + N_8 + N_{10} + N_{12} + N_{14} + N_{16} + N_{18}),$$

$$K_r = e^{-A_2 y} \{ (1 + b_1 A_6) \cos B_2 y - b_1 B_6 \sin B_2 y \} - b_1 A_6 e^{-m_1 y},$$

$$K_i = b_1 B_6 e^{-m_1 y} - e^{-A_2 y} \{ (1 + b_1 A_6) \sin B_2 y + b_1 B_6 \cos B_2 y \},$$

$$L_r = e^{-A_1 y} \left( \cos B_1 y - \frac{4S_c}{\omega} \sin B_1 y \right),$$

$$L_i = \frac{4S_c}{\omega} e^{-S_c y} - e^{-A_1 y} \left( \frac{4S_c}{\omega} \cos B_1 y + \sin B_1 y \right),$$

$$N_1 = b_2 e^{-A_2 y} (A_7 \cos B_2 y - B_7 \sin B_2 y) + b_2 e^{-A_4 y} (B_7 \sin B_4 y - A_7 \cos B_4 y),$$

$$N_2 = b_2 e^{-A_2 y} (B_7 \cos B_2 y - A_7 \sin B_2 y) - b_2 e^{-A_4 y} (A_7 \sin B_4 y + B_7 \cos B_4 y),$$

$$N_3 = b_3 e^{-A_2 y} \{ (A_6 A_7 - B_6 B_7) \cos B_2 y - (A_6 B_7 - B_6 A_7) \sin B_2 y \}$$

$$+ b_3 e^{-A_4 y} \{ (A_6 B_7 - B_6 A_7) \sin B_4 y - (A_6 A_7 - B_6 B_7) \cos B_4 y \},$$

$$N_4 = b_3 e^{-A_2 y} \{ (A_6 A_7 - B_6 B_7) \sin B_2 y + (A_6 B_7 - B_6 A_7) \cos B_2 y \}$$

$$- b_3 e^{-A_4 y} \{ (A_6 A_7 - B_6 B_7) \sin B_4 y + (A_6 B_7 - B_6 A_7) \cos B_4 y \},$$

$$N_5 = b_4 e^{-A_4 y} \{ (A_8 A_6 - B_8 B_6) \cos B_4 y - (A_8 B_6 - B_8 A_6) \sin B_4 y \}$$

$$- b_4 (A_8 A_6 - B_8 B_6) e^{-m_1 y}$$

$$N_6 = b_4 e^{-A_4 y} \{ (A_8 B_6 - B_8 A_6) \cos B_4 y + (A_8 A_6 - B_8 B_6) \sin B_4 y \}$$

$$- b_4 (A_8 B_6 - B_8 A_6) e^{-m_1 y}$$

$$N_7 = b_5 e^{-A_1 y} \left\{ \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) \cos B_1 y - \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) \sin B_1 y \right\}$$

$$+ b_5 e^{-A_4 y} \left\{ \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) \sin B_4 y - \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) \cos B_4 y \right\},$$

$$N_8 = b_5 e^{-A_1 y} \left\{ \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) \cos B_1 y + \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) \sin B_1 y \right\}$$

$$- b_5 e^{-A_4 y} \left\{ \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) \sin B_4 y - \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) \cos B_4 y \right\},$$

$$N_9 = b_6 e^{-A_4 y} (A_{12} \sin B_4 y + B_{12} \cos B_4 y) - b_6 B_{12} e^{-S_c y},$$

$$N_{10} = b_6 A_{12} e^{-S_c y} - b_6 e^{-A_4 y} (A_{12} \cos B_4 y - B_{12} \sin B_4 y),$$

$$N_{11} = b_7 e^{-A_4 y} (A_8 \cos B_4 y - B_8 \sin B_4 y) - b_7 A_8 e^{-m_1 y},$$

$$N_{12} = b_7 e^{-A_4 y} (B_8 \cos B_4 y + A_8 \sin B_4 y) - b_7 B_8 e^{-m_1 y},$$

$$N_{13} = b_8 A_{10} e^{-m_5 y} - b_8 e^{-A_4 y} (A_{10} \cos B_4 y - B_{10} \sin B_4 y),$$

$$N_{14} = b_8 B_{10} e^{-m_5 y} - b_8 e^{-A_4 y} (B_{10} \cos B_4 y + A_{10} \sin B_4 y),$$

$$N_{15} = b_9 e^{-A_4 y} (A_{12} \cos B_4 y - B_{12} \sin B_4 y) - b_9 A_{12} e^{-S_c y},$$

$$N_{16} = b_9 e^{-A_4 y} (B_{12} \cos B_4 y + A_{12} \sin B_4 y) + b_9 B_{12} e^{-m_5 y},$$

$$N_{17} = b_{10} A_{10} e^{-m_5 y} - b_{10} e^{-A_4 y} (A_{10} \cos B_4 y - B_{10} \sin B_4 y),$$

$$N_{18} = b_{10} B_{10} e^{-m_5 y} - b_{10} e^{-A_4 y} (B_{10} \cos B_4 y + A_{10} \sin B_4 y),$$

$$b_1 = \frac{P_r m_1}{(A_6^2 + B_6^2)}, \quad b_2 = \frac{G_r}{(A_7^2 + B_7^2)}, \quad b_3 = \frac{G_r P_r m_1}{(A_6^2 + B_6^2)(A_7^2 + B_7^2)},$$

$$b_4 = \frac{G_r P_r m_1}{(A_6^2 + B_6^2)(A_8^2 + B_8^2)}, \quad b_5 = \frac{G_c}{(A_{11}^2 + B_{11}^2)}, \quad b_6 = \frac{4S_c G_c}{\omega (A_{12}^2 + B_{12}^2)},$$

$$b_7 = \frac{G_r \left( m_1 - \frac{1}{K_p} \right)}{a_4 (A_8^2 + B_8^2)}, \quad b_8 = \frac{G_r \left( m_5 - \frac{1}{K_p} \right)}{a_4 (A_{10}^2 + B_{10}^2)}, \quad b_9 = \frac{G_c \left( -S_c + \frac{1}{K_p} \right)}{a_3 (A_{12}^2 + B_{12}^2)},$$

$$b_{10} = \frac{G_c \left( -m_5 + \frac{1}{K_p} \right)}{a_3 (A_{10}^2 + B_{10}^2)},$$

$$N_r = b_2 A_7 (A_4 - A_2) + b_2 B_7 (B_4 - B_2) + b_3 (A_6 B_7 - B_6 A_7) (B_4 + B_2)$$

$$- b_3 (A_6 A_7 - B_6 B_7) (A_4 + A_2) - b_4 B_4 (A_8 B_6 - B_8 A_6)$$

$$+ b_5 \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) (A_4 - A_1) + b_5 \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) (B_4 - B_1)$$

$$+ B_4 (b_6 A_{12} - b_7 B_8 - b_9 B_{12}) + B_4 B_{10} (b_8 + b_{10})$$

$$+ (m_1 - A_4) [b_4 (A_8 A_6 - B_8 B_6) + b_7 A_8]$$

$$+ (S_c - A_4) (b_6 B_{12} + b_9 A_{12}) + (A_4 - m_5) (b_8 A_{10} + b_{10} A_{10}),$$

$$N_i = b_2 A_7 (B_4 + B_2) + b_3 (A_6 A_7 - B_6 B_7) (B_4 - B_2) - b_4 B_4 (A_8 A_6 - B_8 B_6)$$

$$+ b_5 \left( A_{11} - B_{11} \frac{4S_c}{\omega} \right) (B_4 - B_1) - b_5 \left( B_{11} + A_{11} \frac{4S_c}{\omega} \right) (A_4 - A_1)$$

$$- b_6 A_{12} (A_4 - S_c) - B_4 (b_6 B_{12} + b_7 A_8 + b_9 A_{12}) + B_4 A_{10} (b_8 + b_{10})$$

$$- (A_4 - A_2) [b_2 B_7 + b_3 (A_6 B_7 - B_6 A_7)]$$

$$- (m_1 - A_4) [b_7 B_8 + b_4 (A_8 B_6 - B_8 A_6)]$$

$$- B_{10} (A_4 - m_5) (b_8 + b_{10}) + b_9 B_{12} (m_5 + A_4),$$

$$R_r = A_2 (1 + b_1 A_6) + b_1 (B_2 B_6 - m_1 A_6),$$

$$R_i = B_2 (1 + b_1 A_6) - b_1 (A_2 B_6 + m_1 B_6),$$

$$Q_r = \frac{4S_c}{\omega} B_1 + A_1,$$

$$Q_i = B_1 - A_1 \frac{4S_c}{\omega} + \frac{4S_c^2}{\omega}.$$

## References

- [1] B. Gebhart, L. Pera, The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, *Int. J. Heat Mass Transfer* 14 (12) (1971) 2025–2050.
- [2] K. Gersten, J.F. Gross, Flow and heat transfer along a plane wall with periodic suction, *Z. Angew Math. Phys.* 25 (3) (1974) 399–408.
- [3] V.M. Soundalgekar, S.K. Gupta, Free convection effects on the oscillatory flow past an infinite vertical plate with variable suction and constant heat flux, *Iranian J. Sci. Technol.* 6 (3) (1977) 125–132.
- [4] G.A. Georgantopoulos, J. Koullias, C.L. Goudas, C. Courogenis, Free convection and mass transfer effects on the hydro-magnetic oscillatory flow past an infinite vertical porous plate, *J. Astrophys. Space Sci.* 74 (2) (1981) 357–389.
- [5] T. Hayat, S. Asghar, A.M. Siddiqui, Periodic unsteady flows of a non-Newtonian fluid, *Acta Mechanica* 131 (3–41) (1998) 169–175.
- [6] Y.J. Kim, Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, *Int. J. Eng. Sci.* 38 (8) (2000) 833–845.
- [7] K.D. Singh, R. Sharma, Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability, *Indian J. Pure Appl. Math.* 33 (6) (2002) 941–949.
- [8] A.K. Singh, A.K. Singh, N.P. Singh, Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity, *Indian J. Pure Appl. Math.* 34 (3) (2003) 429–442.
- [9] S. Asghar, M.R. Mohyuddin, T. Hayat, A.M. Siddiqui, The flow of a non-Newtonian fluid induced due to the oscillations of a porous plate, *Math. Probl. Eng.* 2 (2004) 133–143.
- [10] S. Bathul, Heat transfer in three dimensional viscous flow over a porous plate moving with harmonic disturbances, *Bull. Pure Appl. Sci. E* 24 (1) (2005) 69–97.
- [11] P. Singh, C.B. Gupta, MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer, *Indian J. Theor. Phys.* 53 (2) (2005) 111–120.
- [12] S.S. Das, S.K. Sahoo, G.C. Dash, Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction, *Bull. Malays. Math. Sci. Soc.* 29 (1) (2006) 33–42.
- [13] A. Ogulu, J. Prakash, Heat transfer to unsteady magneto-hydrodynamic flow past an infinite moving vertical plate with variable suction, *Phys. Scr.* 74 (2006) 232–239.
- [14] S.S. Das, S.K. Sahoo, G.C. Dash, J.P. Panda, Free convective and mass transfer flow of a viscous incompressible fluid through a porous medium in presence of source/sink with constant suction and heat flux, *AMSE J. Model. Meas. Control B* 75 (2) (2006) 1–20.